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Wave-shaping of pulse tube cryocooler components for improved performance

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ABSTRACT

The method of wave-shaping acoustic resonators is applied to an inertance type cryogenic pulse tube refrigerator (IPTR) to improve its performance. A detailed time-dependent axisymmetric experimentally validated computational fluid dynamic (CFD) model of the PTR is used to predict its performance. The continuity, momentum and energy equations are solved for both the refrigerant gas (helium) and the porous media regions (the regenerator and the three heat-exchangers) in the PTR. An improved representation of heat transfer in the porous media is achieved by employing a thermal non-equilibrium model to couple the gas and solid (porous media) energy equations. The wave-shaped regenerator and pulse tube studied have cone geometries and the effects of different cone angles and the orientation (nozzle v/s diffuser mode) on the system performance are investigated. The resultant spatio-temporal pressure, temperature and velocity fields in the regenerator and pulse tube components are evaluated. The performance of these wave-shaped PTRs is compared to the performance of a non wave-shaped system with cylindrical components. Better cooling is predicted for the cryocooler using wave-shaped components oriented in the diffuser mode.

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1. Introduction and background

The pulse tube refrigerator (PTR) is a type of regenerative refrigerator that runs on gases like helium, nitrogen or air. Being a regenerative system, it runs on a modified reverse Stirling cycle and is capable of achieving cryogenic temperatures. Being a regenerative cryocooler, it also operates with oscillating pressures and mass flows. The oscillating flow is obtained from a pressure wave generator which could be a piston driver (or linear motor) [1,2], an electro-dynamic loudspeaker [3] or a thermoacoustic engine [4–6]. However, given the size of the latter driver mechanism, its application to space cryocooling is challenging.

Unlike the Stirling and Gifford-McMahon (GM) refrigerators, the pulse tube cryocooler does not have a displacer piston [1]. In the Stirling and GM regenerative cryocoolers, the appropriate phasing between the pressure wave and the velocity or mass flow rate is maintained by the displacer piston located in the cold end. However, this moving displacer placed in the cold head leads to vibration in the cold head and possible early failure of the displacer

components. It also requires an additional driver that regulates the displacer's motion which makes the overall system larger. The displacer piston is replaced by a pulse tube, a phase shifting mechanism and a compliance volume. The phase shifting mechanism in a PTR (in lieu of the displacer) is a hollow tube called the inertance tube. Due to the absence of moving parts in the low temperature part of the device, the PTR is suitable for a wide variety of applications and is highly reliable [7]. Such a reliable device has an important role to play in space cryocooling. Pulse tube cryocoolers have been used in industrial applications such as liquefaction of gases [8] and in military applications such as for the cooling of infrared sensors [1]. It has also been suggested that pulse tubes may be used to liquefy oxygen on Mars [9].

The goal of the phasing mechanism is to maintain the pressure and velocity in phase at the center of the regenerator. When the pressure and the velocity are in phase at the center of the regenerator, the mass flow rate (and the resultant pressure drop) in the regenerator is minimized. Various phasing mechanisms have been used in the past [1,10]. Marquardt and Radebaugh reported the highest efficiency for the pulse tube refrigerator when using a combination of the orifice and the inertance tube to control the phase relationships [9].

The term wave-shaping implies the use of non-cylindrical shaped acoustic resonators to improve their output. As opposed to cylindrical resonators, wave-shaped resonators with non-uniform

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cross-sections can have a variety of shapes [11,12]. The phenomenon of wave-shaping was first reported by Lawrenson et al. (experimental) [11] and Ilinskii et al. (numerical) [12] in 1998. The wave-shaped resonators studied in the past have all been standing wave systems [11–13]. The proposed applications of standing wave wave-shaped resonators take advantage of the high frequency, high amplitude and un-shocked pressure waves generated within these systems as well as the strong mixing capabilities (strong streaming velocity fields) [13]. These applications are process reactors, separation or mixing chambers, oil-less compressors and pumps [11]. The field of thermoacoustic cryocooling offers another application that can benefit from the concept of wave-shaping. The need for compact, space worthy cryocoolers has resulted in the need to research high frequency (50–300 Hz) thermoacoustic refrigerators that have high pressure amplitudes. High frequency offers the advantages of a compact system and fast cool-down times from room temperature [7].

There is a difference between the standing wave resonators proposed and the pseudo-travelling wave thermoacoustic PTR systems. In standing wave systems, wave-shaping may only be applied to enhance mixing and generate high pressure amplitude un-shocked waves. However in a PTR, wave-shaping of the components may be harnessed to improve their performance by alternate means due to the fact that the PTR is a pseudo-travelling wave system. These performance enhancements include the suppression of streaming, modifying the pressure and velocity amplitudes and affecting the pressure-velocity phase angle. Acoustic streaming can be defined as steady convection which is driven by oscillatory phenomenon [14] in bounded channels. In a PTR, the function of the pulse tube is to isolate the cold and warm ends from each other. This is accomplished by the formation of a buffer zone in the center of the tube. The occurrence of streaming in the pulse tube leads to a deterioration of its performance due to unwanted mixing of the cold and warm gases which destroys this thermal stratification. Lee et al. [15] first proposed that acoustic streaming can be suppressed in a PTR by using a pulse tube that has a slight taper (which is a form of wave-shaping). In addition to the work by Lee et al., the studies by Olson and Swift [16,17] and Swift et al. [18] showed that by tapering the pulse tube, streaming suppression was achieved. This suppression of streaming resulted in an enhancement in the PTR's performance.

In this study, the concept of wave-shaping is applied to improve the performance of the IPTR. This improvement in performance is achieved by suppressing acoustic streaming in the pulse tube and altering the pressure and velocity amplitudes in the regenerator. In addition to the effects of wave-shaping on the performance, the effect of the wave-shaping on the optimum operating frequency is investigated too. An experimentally validated time-dependent axisymmetric CFD model of the IPTR is used to predict its performance. In the CFD model, the continuity, momentum and energy equations are solved for the fluid (helium gas used as the

refrigerant) and the solid in the regenerator and three heat-exchangers which are modeled as porous media. The details of the model, including problem geometry, boundary and initial conditions and the solution method are discussed in Sections 2 and 3. The results of the computational study are presented in Section 4 and the resultant effects of wave-shaping on the cryocooler performance and quasi-steady phenomena are then discussed.

2. Problem geometry

As stated earlier, the purpose of the study is to investigate the effects of wave-shaping selected components of the PTR on its performance. In order to investigate these effects, the IPTR was studied with both cylindrical and wave-shaped components (regenerator and pulse tube). The straight component geometry is used as the base case against which the performances of the wave-shaped systems are compared. This section discusses the geometry for both these systems.

Fig. 1 shows a schematic of the geometry of the in-line inductance type pulse tube refrigerator (IPTR) simulated. All the components are straight/cylindrical for this geometry.

Table 1 below summarizes the dimensions of the components of the problem geometry (Fig. 1) that do not vary (the same for both the straight and wave-shaped systems). Table 1 also lists the time-invariant boundary conditions for the corresponding components in the simulations. The letters in the first column of Table 1 correspond to the components in Fig. 1. The diameters at the cold and warm ends of the regenerator ($r_{\text{Reg-cold}}$ and $r_{\text{Reg-warm}}$) and the pulse tube ($r_{\text{PT-cold}}$ and $r_{\text{PT-warm}}$) are listed and discussed later.

The IPTR with the wave-shaped components has a geometry similar to that shown in Fig. 1. The main difference for cases with the wave-shaped geometry is diameter at the ends of the wave-shaped component. Fig. 2 below shows a schematic of the various wave-shaped regenerators simulated. The components shown in the figure correspond to the aftercooler heat-exchanger (C), the regenerator (D) and the cold heat-exchanger (E) as seen in Fig. 1 and Table 1. The schematics in Fig. 2a, Fig. 2b and Fig. 2c correspond to the straight regenerator, the diffuser mode regenerator (negative taper angle) and the nozzle mode regenerator (positive taper angle), all drawn to scale. As can be seen in the figure (2a–c) below, the aftercooler heat-exchanger diameter is always maintained the same as the diameter of the warm end of the regenerator ($r_{\text{Reg-warm}}$). The diameter of the cold heat-exchanger is not dependent on the diameter of the cold end of the regenerator ($r_{\text{Reg-cold}}$), but on the diameter of the cold end of the pulse tube ($r_{\text{PT-cold}}$). The diameter of the transfer tube component (B, not shown in Fig. 2) is maintained the same as the diameter of the warm end of the regenerator (and the aftercooler heat-exchanger).

A similar convention is followed when the pulse tube component is wave-shaped (Fig. 3 below, note that the taper is

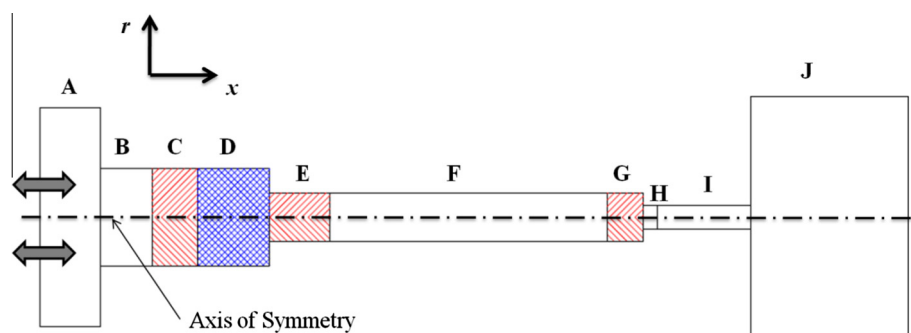


Fig. 1. A schematic of the in-line inductance type pulse tube refrigerator simulated (all components straight/cylindrical).

Table 1
Dimensions of the IPTR system simulated and time invariant thermal boundary conditions.

No.	Component	Radius (cm)	Length (cm)	Thermal boundary condition
A	Compression chamber	3.00	1.1	Adiabatic
B	Transfer tube	$r_{\text{Reg-warm}}^a$	4.0	$h_c = 20 \text{ W/m}^2 \text{ K}$
C	Aftercooler heat-exchanger	$r_{\text{Reg-warm}}^a$	3.0	$T_w = 293 \text{ K}$
D	Regenerator	$-^a$	6.0	Adiabatic
E	Cold heat-exchanger	$r_{\text{PT-cold}}^a$	5.0	Adiabatic
F	Pulse tube	$-^a$	23.0	Adiabatic
G	Hot heat-exchanger	$r_{\text{PT-warm}}^a$	3.0	$T_w = 293 \text{ K}$
H	Connector	0.1524	0.4	Adiabatic
I	Inertance tube	0.1524	170.70	Adiabatic
J	Compliance volume	2.60	14.9	Adiabatic

^a See Table 3.

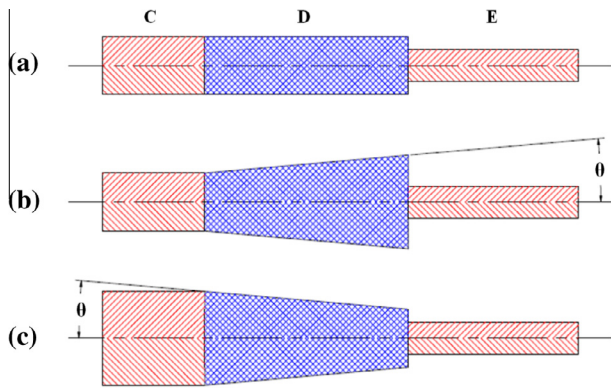


Fig. 2. Schematics (to scale) of (a) the straight regenerator and the wave-shaped regenerators: (b) diffuser mode and (c) nozzle mode.

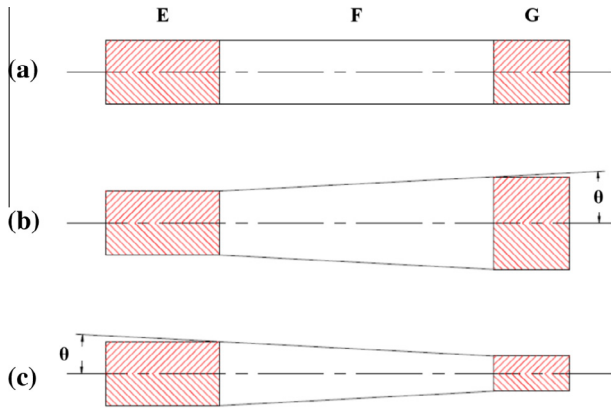


Fig. 3. Schematics of (a) the straight pulse tube and the wave shaped pulse tubes: (b) the diffuser mode and (c) the nozzle mode.

exaggerated and not drawn to scale). The schematics in Fig. 3a–c correspond to the straight pulse tube, the diffuser mode pulse tube and the nozzle mode pulse tube. The components shown in Fig. 3 are the cold heat-exchanger (E), the pulse tube (F) and the hot heat-exchanger. The diameters of the cold and hot heat-exchangers are always maintained the same as the diameters of the cold ($r_{\text{PT-cold}}$) and warm ($r_{\text{PT-warm}}$) ends of the pulse tube respectively. As stated earlier, the sign convention for the taper angles is positive for the nozzle mode and negative for the diffuser mode.

3. Mathematical model

The simulation model of the cryogenic pulse tube refrigerator incorporates the fluid dynamic equations of conservation of mass

(continuity equation), momentum (Navier–Stokes equations) and energy in the computational domain. The various heat exchangers and the regenerator are modeled as porous media with the relevant solid properties of the regenerator and the heat exchanger materials. The porous media model accounts for the non-equilibrium heat-transfer between the gas and the solid (porous media) phases. The mass, momentum and energy conservation equations [19–21] for the porous media regions are solved simultaneously with the corresponding gas phase equations for the IPTR as shown in Fig. 1.

3.1. Governing equations

The thermo-fluidic conservation equations for the refrigerant gas (helium) within the cryogenic pulse tube refrigeration system are given as follows:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \vec{u}) = 0 \quad (1)$$

$$\frac{\partial (\rho \vec{u})}{\partial t} + \nabla \cdot (\rho \vec{u} \vec{u}) = -\nabla p + \nabla \cdot \tau_{ij} \quad (2)$$

$$\frac{\partial (\varepsilon \rho h_0)}{\partial t} + \nabla \cdot (\varepsilon \rho \vec{u} h_0) = \nabla \cdot (k_f \nabla T_f) + \nabla \cdot (\varepsilon \vec{u} \tau_{ij}) + \varepsilon \frac{\partial p}{\partial t} + S_f \quad (3)$$

$$p = \rho R T_f \quad (4)$$

where ρ is the density, $\vec{u}$ is the $(r - x)$ velocity vector, p is the pressure, τ_{ij} is the viscous stress tensor, T_f is the gas/fluid temperature, R is the ideal gas constant, k_f is the thermal conductivity of the fluid, ε is the porosity of the porous substance ($\varepsilon = 1.0$ for non-porous zones) and the total enthalpy is h_0 given by:

$$h_0 = i + \frac{p}{\rho} + \frac{1}{2} (\vec{u})^2$$

$$i = c T_f \quad (5)$$

$$T_f = \frac{1}{c} \left[h_0 - \frac{p}{\rho} - \frac{1}{2} (\vec{u})^2 \right]$$

where i is the internal energy and c is the specific heat capacity of the fluid. The source term S_f in the fluid energy equation (Eq. (3)) is only applicable for the porous media zones (where the porosity ε is < 1.0):

$$S_f = \frac{h_{fp} A (T_p - T_f)}{\varepsilon} \quad (6)$$

where T_p is the temperature of the solid in the porous media zones. The value of the convective heat transfer coefficient h_{fp} for the gas–solid interfaces is calculated from a heat transfer correlation for oscillating flow in regenerators developed by Tanaka et al. [22]. The equation is given by:

$$h_{fp} = 0.33 \frac{k_f}{4r_h} \left(\frac{8\rho_f r_h V_p \rho_r}{\mu A_{s,p} \varepsilon} \right)^{0.67} \quad (7)$$

where r_h is the hydraulic radius of the porous media material (for wire mesh, r_h is dependent on the porosity and the diameter of the wire), V_p is the volume of the solid material in the porous media, $A_{s,p}$ is the total surface area of the porous media, μ is the viscosity of the fluid and ρ_r is the ratio of the density of the fluid to the average density of the fluid in the porous media.

The refrigerant helium is assumed to be an ideal gas and its viscosity and thermal conductivity are considered to be temperature-dependent. The temperature dependent properties were obtained from the NIST database [23]. The specific heat was kept constant.

The mass and momentum equations for the porous media zones (Darcy–Forchheimer model) [21,24] are given as follows:

$$\frac{\partial(\varepsilon\rho)}{\partial t} + \nabla \cdot (\varepsilon\rho\vec{u}) = 0 \quad (8)$$

$$\frac{\partial(\varepsilon\rho\vec{u})}{\partial t} + \nabla \cdot (\varepsilon\rho\vec{u}\vec{u}) = -\varepsilon\nabla p + \nabla \cdot (\varepsilon\tau_{ij}) - \frac{\varepsilon^2\mu}{\kappa}\vec{u} - \frac{\varepsilon^3 C_F \rho}{\sqrt{\kappa}}|\vec{u}|\vec{u} \quad (9)$$

Similar to the porosity, the permeability κ and quadratic drag factor C_F have values that are dependent on the component and are listed in Table 2 below.

For the solid matrix, the energy equation is given by:

$$\frac{\partial(T_p)}{\partial t} = \nabla \cdot (\Gamma\nabla T_p) + S_p \quad (10)$$

where Γ is the diffusivity of the solid porous media and S_p is the source term that couples the solid/porous media energy equation (Eq. (10)) with the fluid/gas energy equation (Eq. (3)) through the corresponding gas-energy equation source term S_f (Eq. (6)). This source term S_p is given by:

$$S_p = \frac{h_{fp}A(T_f - T_p)}{1 - \varepsilon} \quad (11)$$

where A is the heat transfer area per unit volume. The value of Γ for copper (in the heat exchangers) is $1.17 \times 10^{-4} \text{ m}^2/\text{s}$ and that for stainless steel (in the regenerator) is $1.18 \times 10^{-5} \text{ m}^2/\text{s}$.

4.2. Initial conditions

The initial pressure in the system is set to 2.2 MPa for all the cases. In order to predict the system's performance near quasi-equilibrium, the computational model was run with a prescribed initial condition that represents a “near quasi-steady” temperature distribution. The cold heat-exchanger is assumed to be at 100 K, the aftercooler and warm heat-exchangers at 300 K and a linear gradient is assumed across the lengths of the regenerator and pulse tube. With this initial condition, the simulations were run until a quasi-stead state was achieved.

4.3. Boundary conditions

Table 1 (given earlier) specifies the thermal boundary conditions used at the surface boundaries of the various components.

Table 2
Porous media parameters used in the simulation.

Component	Material	Porosity	Permeability (m ²)	Drag factor
Heat-exchangers	Copper	0.774	4.08×10^{-08}	0.20
Regenerator	Stainless steel	0.720	9.70×10^{-11}	0.30

The piston is modeled as a reciprocating wall having an oscillatory velocity. The velocity of the piston is defined by the function, $u = A_0 \omega \cos(\omega t)$, where A_0 is the maximum displacement of the piston (piston amplitude), ω is the angular frequency ($\omega = 2\pi f$) and f is the frequency of operation. For the current simulations, the piston amplitude A_0 is set as 1.75 mm. The frequency was varied in the system and the values of the frequency simulated are discussed later.

4.4. Porous media conditions

In the porous media regions, the three heat exchangers (components C, E and G in Fig. 1) have properties of copper with density 8950 kg/m³, specific heat 380 J/kg K and thermal conductivity of 470 W/m K. The regenerator (component D in Fig. 1) has properties of 316 stainless steel, with density 7810 kg/m³, specific heat 460 J/kg K and thermal conductivity of 10 W/m K.

Table 2 lists the porous media parameters used in the governing equations of the porous media. The values for porosity, permeability and drag factor are the same as those used in a previous study where the cryocooler model was validated against temporal and quasi-steady experimental results [25].

4.5. Numerical scheme

The numerical scheme for solving the governing equations is based on the finite volume approach. The continuity, momentum and energy equations are solved using the 2nd Order Upwind difference scheme. The motion of the piston is captured by a moving grid scheme near the piston wall (component A, Fig. 2). The remeshing scheme used for the moving/deforming mesh is the Transfinite Interpolation scheme [21].

A 2nd order Crank–Nicholson scheme (with a blending factor of 0.7) is used for the time derivatives in the continuity, momentum and energy equations. The time-step (Δt) for the simulations was dependent on the frequency studied. This value of ‘ Δt ’ was assumed such that each cycle was discretized into 160 time instants. An overall convergence criterion is set for all the variables at 10^{-4} in the iterative implicit numerical solver. Due to the symmetry of the problem geometry, only one-half of the PTR's geometry (the “Axis of Symmetry” line is defined in Fig. 1) was considered for the simulations. The total number of grid points in the system was 10533. A hybrid meshing scheme was used with unstructured mesh in the cold heat-exchanger due to the high temperature gradient observed there. The governing equations and the boundary conditions were solved using CFD-ACE+ [21].

5. Results

The transient and quasi-steady state results of the simulations are presented and described in the following section. The significance of the effects of the taper angle of the regenerator and the pulse tube as well as the IPTTR's operating frequency on the system's near-equilibrium quasi-steady performance are also discussed.

The cases simulated are listed in Table 3 below. The first five cases correspond to the wave-shaped regenerator (Fig. 2a–c) and the pulse tube is maintained cylindrical/straight. Cases 6 and 7 correspond to the wave-shaped pulse tube components (Fig. 3b and c), however the regenerator is maintained at the optimum taper observed (i.e., case 2, diffuser mode). For all the wave-shaped component cases, only the taper angle is changed. The volume and the length of each component are maintained constant and in order to change the taper angle, the radii at the opposite ends of the component are varied.

Table 3

List of cases simulated.

Case no.	Frequency (Hz)	Taper angle regenerator θ_R (°)	Taper angle pulse tube θ_{PT} (°)	Radius- regenerator warm end (cm)	Radius- regenerator cold end (cm)	Radius- pulse tube warm end (cm)	Radius- pulse tube cold end (cm)
1	65	-4.76	0.000	0.60	1.10	0.47	0.47
2	65	-2.90	0.000	0.70	1.00	0.47	0.47
3	65	0.00	0.000	0.85	0.85	0.47	0.47
4	65	2.90	0.000	1.00	0.70	0.47	0.47
5	65	4.76	0.000	1.10	0.60	0.47	0.47
6	65	-2.90	-0.598	0.70	1.00	0.35	0.59
7	65	-2.90	0.598	0.70	1.00	0.59	0.35
8	60	-2.90	-0.598	0.70	1.00	0.35	0.59
9	63	-2.90	-0.598	0.70	1.00	0.35	0.59
10	67	-2.90	-0.598	0.70	1.00	0.35	0.59
11	70	-2.90	-0.598	0.70	1.00	0.35	0.59

Finally, wave-shaping of resonators is known to cause a change in the resonant frequency, i.e., resonators with similar lengths have different resonant frequencies depending on the shape of the resonator [11–13]. However, the systems studied were purely standing wave systems. The IPTR is a pseudo-travelling wave system by virtue of the compliance volume (component J) which prevents large reflections of the incident wave. In order to confirm the optimum operating frequency of the cryocooler after the components were wave-shaped, the last four cases were studied. In these cases (cases 8–11), the frequency for the system with the best performance (system comprised of two wave-shaped components) was varied to check for changes in the performance (i.e., to find a new optimum operating frequency if it exists).

5.1. Temporal evaluation of the IPTR's performance

The cool-down characteristic of a PTR exhibits the transient response of the system and provides an important indication of the PTR's performance. In Fig. 4 below, the cool-down characteristics of the IPTR are shown for cases 1–5 where the regenerator taper angle is investigated. As can be seen in the figure, the performance of the IPTR marginally improves for the system with the regenerator in the diffuser mode and degrades for the regenerator in the nozzle mode. The cool-down characteristics indicate that

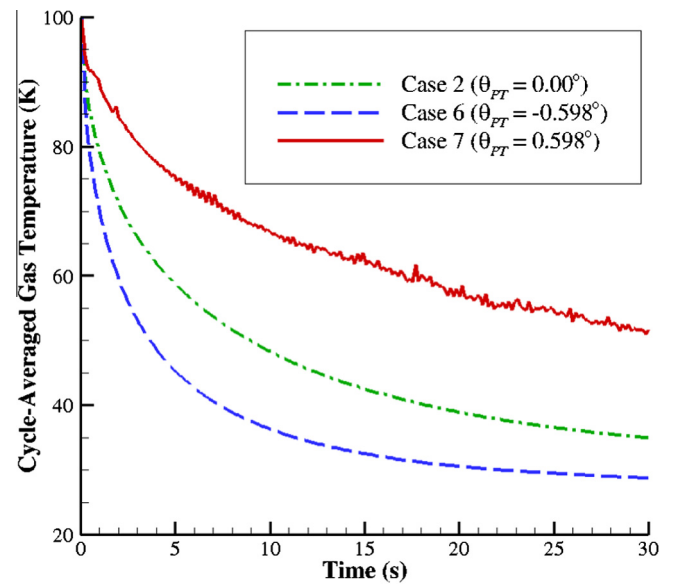


Fig. 5. Transient variation of the cycle-averaged gas temperature at the exit of the cold heat-exchanger from the near quasi-steady initial condition comparing the performance of the PTR for the wave-shaped pulse tube.

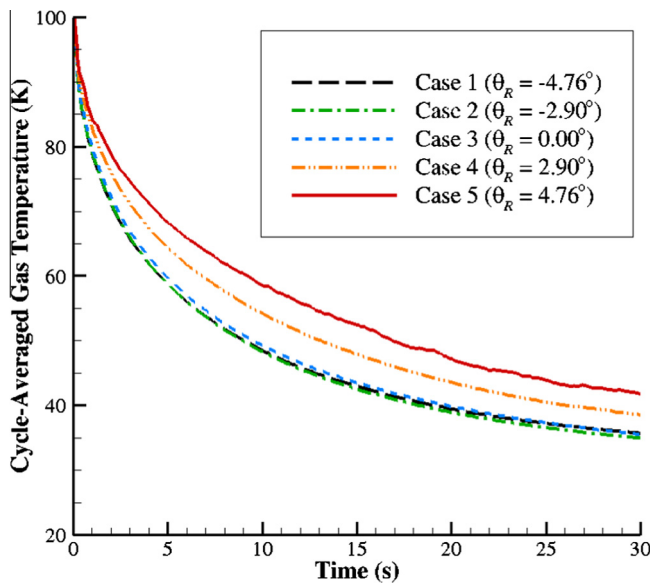


Fig. 4. Transient variation of the cycle-averaged gas temperature at the exit of the cold heat-exchanger from the near quasi-steady initial condition for cases 1–5 (Note: cases 1, 2 and 3 show similar performance).

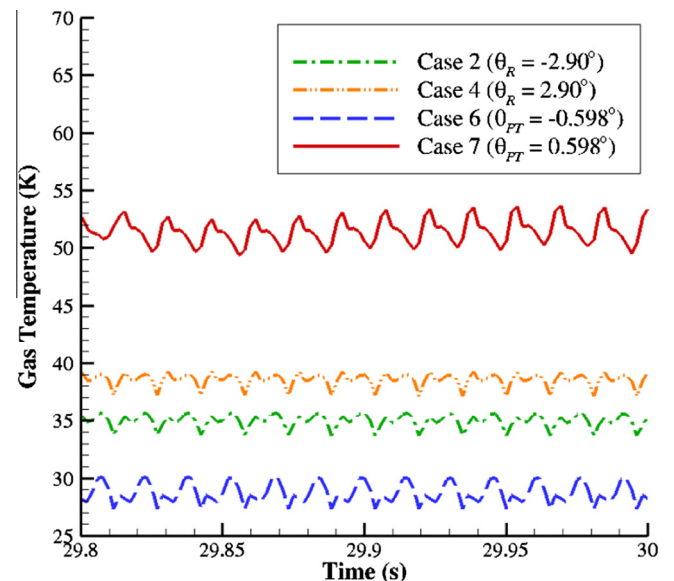


Fig. 6. Near quasi-equilibrium variation of the gas temperature at the exit of the cold heat-exchanger ($x = 19.1$ cm, $r = 0.0$ cm).

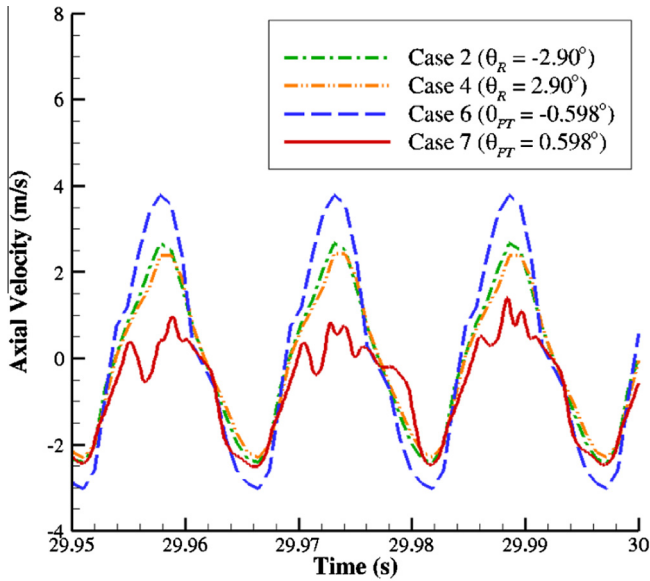


Fig. 7. Near equilibrium variation in the gas velocity at the entrance of the pulse tube ($x = 19.1$ cm, $r = 0.0$ cm).

case 2 ($\theta_R = -2.90^\circ$) has the best performance and case 5 ($\theta_R = 4.76^\circ$) has the worst performance. From Fig. 4 it can be summarized that the taper of the regenerator does not have much of an effect on the IPTR in the configuration and for the operating conditions studied.

Due to the minor variations in performance observed when the IPTR's regenerator is wave-shaped, the effect of wave-shaping the pulse tube was investigated. To investigate the wave-shaping of the pulse tube, the optimum geometry observed for the wave-shaped regenerator cases (lowest temperature) was used. Fig. 5 shows the cool-down characteristics of the IPTR for cases 2, 6 and 7, which correspond to pulse tube taper angles θ_{PT} of 0.0° , -0.598° and 0.598° . Two observations are apparent from the figure. The first is that much larger variations in performance are observed for the wave-shaped pulse tube than the wave-shaped regenerator and at smaller taper angles. The second is that even the cycle-averaged temperature variations for case 7 ($\theta_{PT} = 0.598^\circ$) have large fluctuations from cycle to cycle.

The results shown in Figs. 4 and 5 are cycle-averaged, however due to the cyclic nature of the flow in the cryocooler, the temperature oscillates too. Fig. 6 shows the oscillatory temperature profiles at the exit of the cold heat-exchanger. The sinusoidal pressure variations in the system should correspond to sinusoidal temperature variations (due to the ideal gas assumption). However, due to the acoustic streaming phenomenon observed in the pulse tube [25–27], the oscillatory temperature profiles have distorted sinusoidal shapes as seen in Fig. 6. This acoustic streaming arises due to the non-zero cycle averaged velocity in the pulse tube component and will be discussed further in the following section. Even though acoustic streaming occurs, near quasi-steady state the variation in the temperature profiles from cycle to cycle are small. This is due to the fact that the velocity profiles are consistent from cycle to cycle (Fig. 7). The exception to this is observed in case 7 where the pulse tube is in the nozzle mode. As was observed in Fig. 5, the cycle-averaged temperature profile for case 7 is not smooth. This indicates that there is larger scale temperature oscillation observed at the exit of the cold heat-exchanger for case 7. This can be observed in Fig. 6 too where the oscillatory temperature profiles show some variation from cycle to cycle even near quasi-steady state. This cycle to cycle variation in the profiles for case 7 is observed for the axial component of the velocity too as seen in Fig. 7. The significance of this variation is discussed below in the following section with respect to the acoustic streaming observed in the pulse tube component.

Fig. 8 below shows plots of the temporal variations of the axial component of the gas velocity and the pressure as a function of the cycle time near quasi-steady state for case 2 (Fig. 8a) and case 6 (Fig. 8b) at the midpoint of the regenerator ($x = 11.1$ cm, $r = 0.0$ cm). Since the inertance in the system is an optimized value, the phase angle between the velocity and the pressure is close to zero for both the cases. Wave-shaping of the regenerator section affects the flow fields (pressure and velocity amplitudes) in the regenerator itself as expected (Fig. 9a). Even though the regenerator is comprised of porous media, the variations in velocity amplitude at the midpoint of the regenerator are large when it is changed from the diffuser mode to the nozzle mode. However, a more surprising result is that a minor taper of the pulse tube ($\theta_{PT} < 1^\circ$) causes changes in the velocity amplitude at the center of the regenerator (Fig. 9b). An important point of note is that since the regenerator has been tapered/wave-shaped, the location where the pressure and velocity are in phase may not be the center of the regenerator.

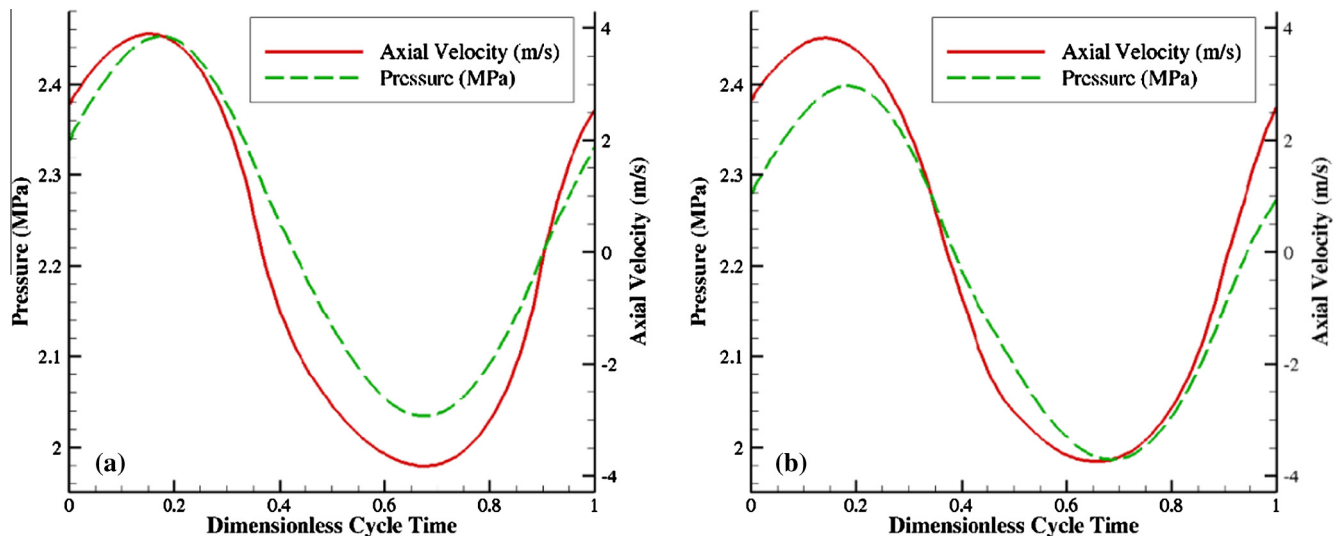


Fig. 8. Temporal variation of the pressure and axial component of the velocity at the midpoint of the regenerator ($x = 11.1$ cm, $r = 0.0$ cm) for (a) case 2 (regenerator taper) and (b) case 6 (regenerator and pulse tube taper).

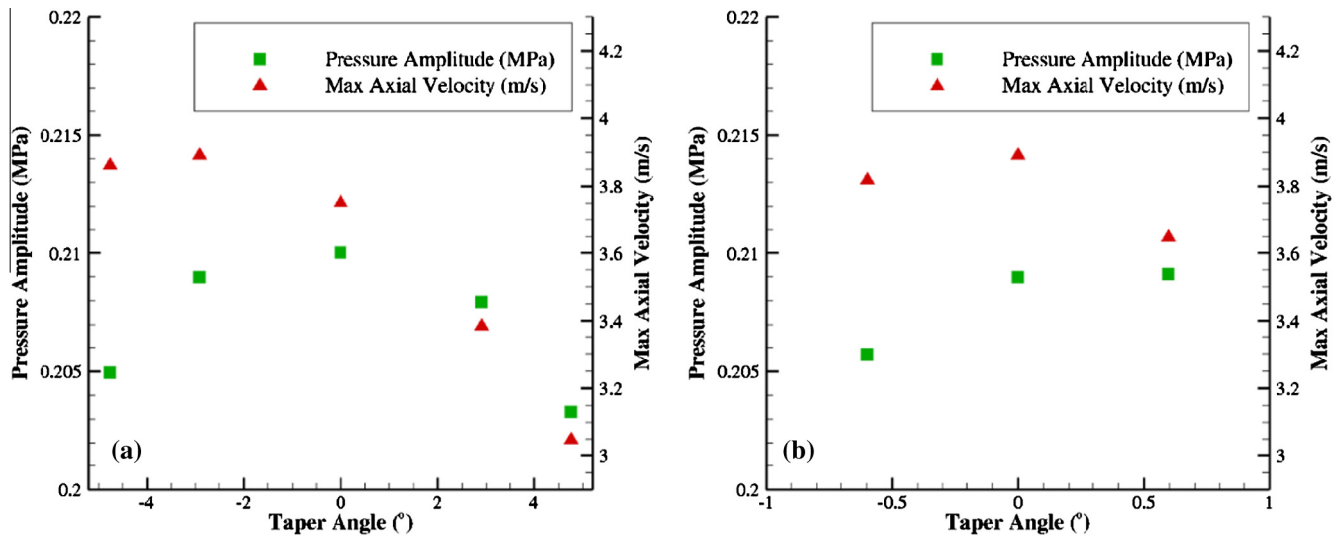


Fig. 9. Pressure amplitude and amplitude of the axial component of the velocity at the midpoint of the regenerator ($x = 11.1$ cm, $r = 0.0$ cm) as a function of the taper angle of the (a) regenerator and (b) pulse tube.

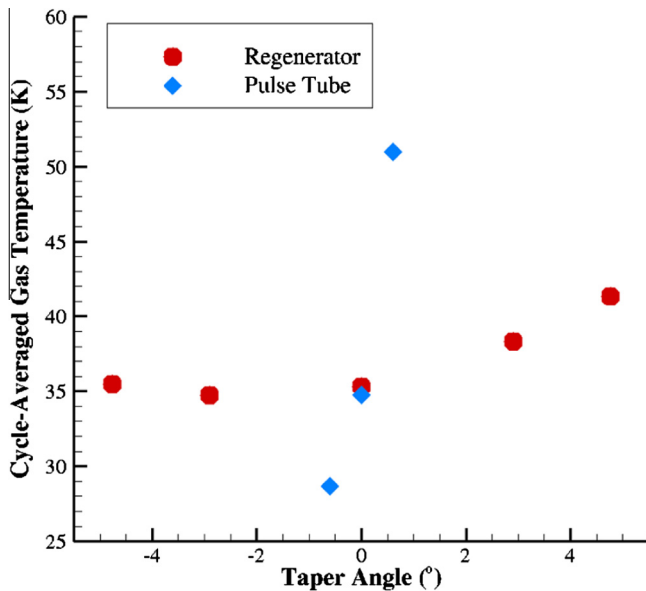


Fig. 10. Performance map (near equilibrium cycle-averaged gas temperature v/s taper angle) of the PTR for the wave-shaped geometries studied.

Comparing the results for the first seven cases studied, it can be concluded that the regenerator taper does not result in large changes in the cryocooler performance. This is summarized graphically in Fig. 10 where the cycle-averaged gas temperature at the exit of the cold heat-exchanger is plotted as a function of the taper angle for both the regenerator and the pulse tube. Larger changes in performance are observed when the pulse tube component is wave-shaped. It is interesting to note that these minor changes in the pulse tube taper angle lead to such large variations in quasi-steady state gas temperature.

Since the IPTR is a thermoacoustic system, the operating frequency affects the performance too [25,27]. As has been seen in the past [11–13], wave-shaping of resonators affected the optimum/resonant frequency. To study the effects of operating frequency on the system's performance, the case with the optimum geometry/lowest gas temperature was used (i.e., case 6). The operating frequency was varied about 65 Hz which was the optimum for the straight system. The cycle-averaged gas temperature at

the exit of the cold heat-exchanger for these is plotted in Fig. 11 below. As can be seen from this figure, the optimum operating frequency is still 65 Hz even though both the regenerator and the pulse tube were tapered (both in the diffuser mode). However, it is also interesting to note that there is no major effect of the operating frequency on the IPTR's performance.

5.2. Effects of wave-shaping on the quasi-steady phenomena in the PTR

The transient results discussed above show interesting trends in the performance of the IPTR for the different wave-shaped geometries studied. However, the large changes observed in the performance of the system with the wave-shaped pulse tube cannot be explained by the transient results shown above (pressure and velocity profiles and the phase angles between the two). Hence in this section the quasi-steady phenomena in the IPTR are investigated.

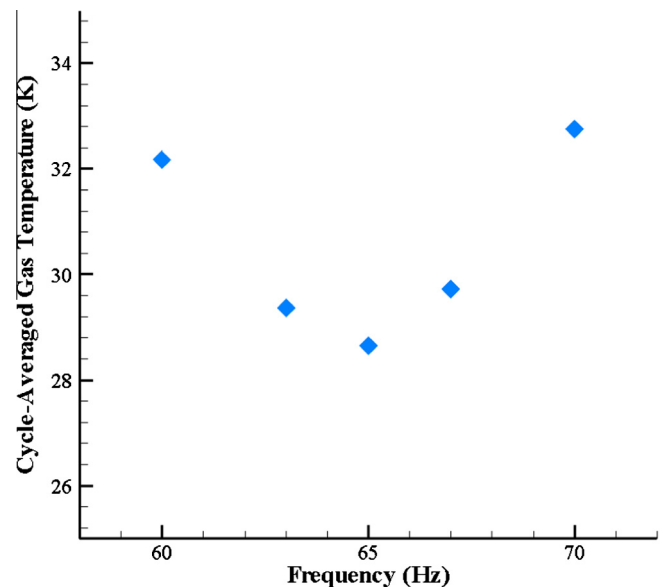


Fig. 11. Variation of the cycle-averaged gas temperature at the exit of the cold heat-exchanger as a function of the driver frequency.

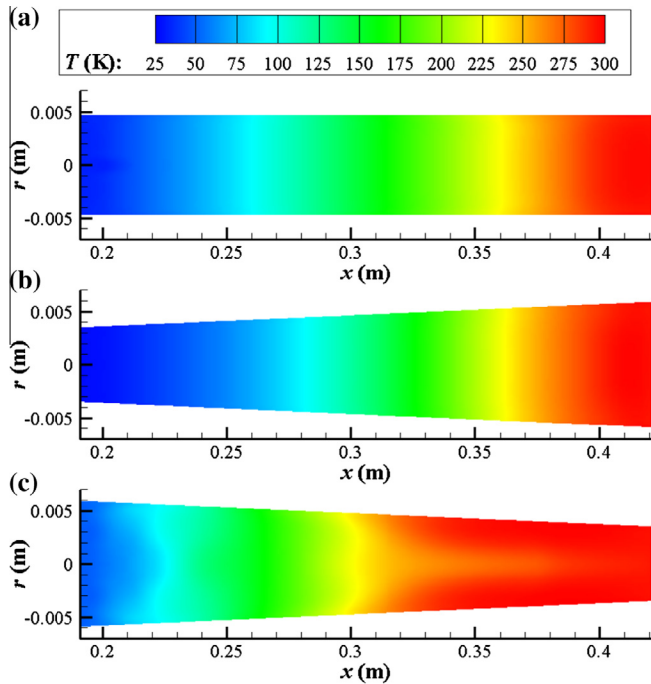


Fig. 12. Cycle-averaged gas temperature contours in the pulse tube section for the 2001st cycle comparing the (a) straight mode-case 2, (b) diffuser mode-case 6 and (c) nozzle mode-case 7 geometries.

The cycle-averaged gas temperature in the pulse tube section of the IPTR is plotted for the 2001st cycle (near quasi-steady state) in Fig. 12 below. Fig. 12a–c are for cases 2, 6 and 7 respectively. The axial variation in the gas temperature for cases 2 and 6 (Fig. 12a and b respectively) appear to be one-dimensional in nature with negligible radial variation. This indicates that the pulse tube has near perfect thermal stratification for these cases and that steady acoustic streaming has a minimal effect on the performance of the system. This is further evidenced by the vector plots in Fig. 13a and b (cases 2 and 6 respectively) where the Favre-aver-

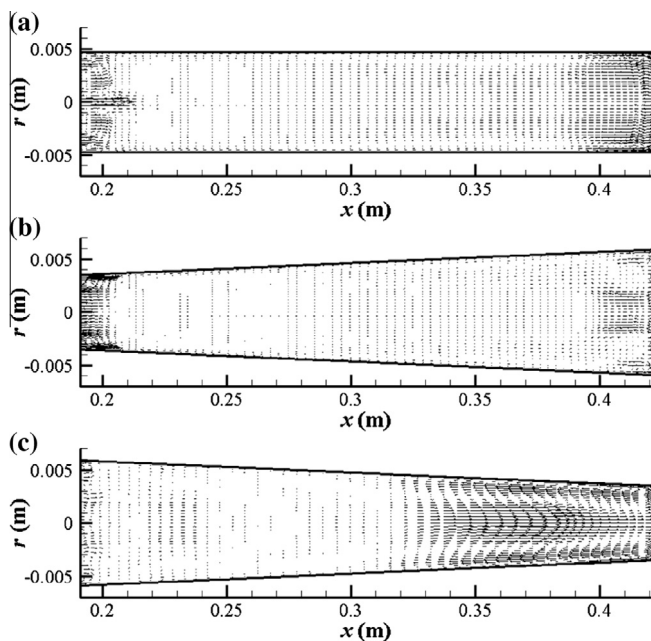


Fig. 13. Cycle-averaged (Favre-averaged) gas velocity vectors in the pulse tube section for the 2001st cycle comparing the (a) straight mode-case 2, (b) diffuser mode-case 6 and (c) nozzle mode-case 7 geometries.

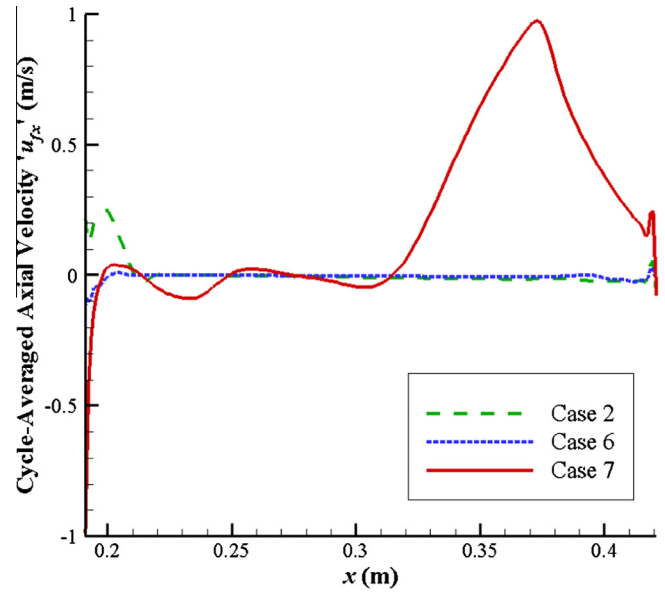


Fig. 14. Axial variation of the Favre-averaged gas velocity in the pulse tube section along the axis of symmetry ($r = 0.0$) for the wave-shaped pulse tube.

aged [28] gas velocity is plotted in the pulse tube component for the same cycle (# 2001). In Fig. 13a and b, a vector of length 1.0 cm corresponds to a velocity of 0.25 m/s. For Fig. 13c, a vector length of 1.0 cm corresponds to a velocity of 1.0 m/s.

The acoustic streaming vectors in Fig. 13a and b are very small (indicating low streaming velocities) except near the cold end of the pulse tube. For case 6 (diffuser mode of the pulse tube, Fig. 13b) there are sections of the pulse tube where there is no acoustic streaming flow ($u_{fx} \sim 10^{-3} - 10^{-4}$ m/s). This explains why the pulse tube is thermally stratified for cases 2 and 6 and why the performance is better than case 7. Comparing Fig. 12c (case 7) to Fig. 12a and b (cases 2 and 6) it is clear that there exists a considerable acoustic streaming flow that is affecting the flow in the pulse tube and thus affecting the temperature fields. This is also seen in Fig. 13c where the acoustic streaming velocities are very high in the warm end of the pulse tube. This acoustic streaming field is also stronger in the rest of the pulse tube compared to cases 2 and 6 (Fig. 13a and b). Such high acoustic streaming fields cause mixing and re-circulation of the flow in the pulse tube thereby deteriorating the performance.

The axial component of the Favre-averaged gas velocity at the axis of symmetry is plotted as a function of the pulse tube length in Fig. 14 for cases 2, 6 and 7. The negative values indicate flow in the negative x -direction. The profiles for cases 2 and 6 are similar except near the cold end of the pulse tube where the acoustic streaming velocity is larger for case 2. This suggests that the performance of the IPTR is better for case 6 (which was observed in Fig. 10) due to lower mixing/re-circulation of the gas in the cold end of the pulse tube for case 6. Similar to the vector plot in Fig. 13c, the acoustic streaming velocity for case 7 is very high ($u_{fx} \sim \pm 1.0$ m/s) in the cold and warm ends of the tube. This indicates large re-circulation of the cold and warm gas in the pulse tube section and explains why the performance of the pulse tube in the nozzle mode is the most un-favorable.

6. Conclusions

The numerical study reported in this paper employs the method of wave-shaping to improve the performance of an inertance type PTR. The transient numerical model solves the governing equations of fluid mechanics and heat transfer for both the fluid (refrigerant

gas) and the solid/porous (heat-exchanger mesh) media in the domain. Two components in the PTR are wave-shaped: the regenerator and the pulse tube. The wave-shaped geometry studied is a cone and the effects of both the diffuser and nozzle orientations are investigated.

For the wave-shaped regenerator, the quasi-steady state gas temperature in the cold end does not show much variation for the geometries studied ($\sim 2\text{--}5\text{ K}$). However, wave-shaping of the regenerator leads to changes in the pressure and velocity amplitudes in the regenerator. Most importantly, wave-shaping the regenerator shifts the location in the regenerator where the pressure and velocity waves are in phase (it is in the center of a straight regenerator for an optimum inertance value). Wave-shaping the pulse tube has a more pronounced effect on the performance (gas temperature in the cold end) of the PTR. A small taper variation (θ_{PT}) from -0.598° to 0.598° results in a temperature variation of about 25 K. For the pulse tube component in the diffuser mode, the temperature attained in the cold end near quasi-steady state is the lowest. Additionally, the cool-down time for the diffuser mode tapered pulse tube is much shorter compared to the straight pulse tube (the magnitude of the slope during the initial cool-down is higher). These improvements in performance are due to a reduction in acoustic streaming and mixing in the pulse tube section for the diffuser mode geometry. These low strength acoustic streaming fields do not affect the temperature profiles in the pulse tube. Hence near one-dimensional temperature profiles and thermal stratification exists in the pulse tube. For the pulse tube in the nozzle taper mode, oscillations are observed in the cycle-averaged temperature too. These oscillations are attributed to variations in the temporal velocity profiles in the pulse tube section. In addition to the transient distorted temperature oscillations, the strength of the steady acoustic streaming field in the pulse tube is very high. These high strength acoustic streaming patterns also affect the cycle averaged temperature profiles in the pulse tube section and thermal stratification is not observed for this case. Finally, the frequency of the driver was varied and the effect of the operating frequency on the PTR's performance was investigated. Unlike standing wave resonators studied in the past, the PTR is a quasi-travelling wave system and hence wave-shaping the components did not affect the optimum operating frequency. The optimum operating frequency was 65 Hz for the components and inertance studied. The overall conclusion of the numerical study is that wave-shaping of the pulse tube component has a major effect on the performance of a PTR. This change in the performance is largely due to the effect of wave-shaping on the acoustic streaming patterns in the pulse tube component.

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References

- [1] Radebaugh R. Pulse tube cryocoolers for cooling infrared sensors. San Diego (CA, USA): Society of Photo-Optical Instrumentation Engineers; 2000. p. 363–79.
- [2] Radebaugh R, Garaway I, Veprik AM. Development of miniature, high frequency pulse tube cryocoolers. Infrared technology and applications XXXVI, USA: SPIE – The International Society for Optical Engineering; 2010. p. 76602J (14 pp.).
- [3] Fan L, Zhang S-Y, Zhang H. Nonlinear effects in thermoacoustic refrigerators driven by voltage-excited loudspeakers. *J Appl Phys* 2008;104(11).
- [4] Hatori H, Biwa T, Yazaki T. How to build a loaded thermoacoustic engine. *J Appl Phys* 2012;111(7).
- [5] Ercang L, Chen Y, Wei D. Heat transfer characteristics of oscillating flow regenerator filled with circular tubes or parallel plates. *Cryogenics* 2007;47(1):40–8.
- [6] Wei D, Ercang L, Yong Z, Hong L. Detailed study of a traveling wave thermoacoustic refrigerator driven by a traveling wave thermoacoustic engine. *J Acoust Soc Am* 2006;119(5):2686–92.
- [7] Radebaugh R. Cryocoolers: the state of the art and recent developments. *J Phys: Condens Matter* 2009;21(16):164219.
- [8] Swift GW. Thermoacoustic natural gas liquefier. In: DOE natural gas conference. Houston, TX; 1997.
- [9] Marquardt ED, Radebaugh R. Pulse tube oxygen liquefier. *Adv Cryog Eng* 2000;45457–64.
- [10] Iwase T, Biwa T, Yazaki T. Acoustic impedance measurements of pulse tube refrigerators. *J Appl Phys* 2010;107(3):034903–34906.
- [11] Lawrenson CC, Lipkens B, Lucas TS, Perkins DK, Van Doren TW. Measurements of macrosonic standing waves in oscillating closed cavities. *J Acoust Soc Am* 1998;104(2):623–36.
- [12] Ilinskii YA, Lipkens B, Lucas TS, Van Doren TW, Zabolotskaya EA. Nonlinear standing waves in an acoustical resonator. *J Acoust Soc Am* 1998;104(Copyright 1998, IEEE):2664–74.
- [13] Antao DS, Farouk B. High amplitude nonlinear acoustic wave driven flow fields in cylindrical and conical resonators. *J Acoust Soc Am* 2013 (in press).
- [14] Nyborg WL. Acoustic streaming. In: Hamilton MF, Blackstock DT, editors. *Nonlinear acoustics*. Academic Press; 1998. p. 207–31.
- [15] Lee JM, Kittel P, Timmerhaus KD, Radebaugh R. Flow patterns intrinsic to the pulse tube refrigerator. In: Ross RG, editor. 7th International cryocooler conference. Kirtland Air Force Base (NM): Phillips Laboratory; 1993. p. 125–39.
- [16] Olson JR, Swift GW. Acoustic streaming in pulse tube refrigerators: tapered pulse tubes. *Cryogenics* 1997;37(Compendex):769–76.
- [17] Olson JR, Swift GW. Suppression of acoustic streaming in tapered pulse tubes. In: Ross RG, editor. 10th International cryocooler conference. Monterey (CA): Plenum Publishers; 1999.
- [18] Swift GW, Allen MS, Wollan JJ. Performance of a tapered pulse tube. In: Ross RG, editor. 10th International cryocooler conference. Monterey (CA): Plenum Publishers; 1999.
- [19] Wang CY, Gu WB, Liaw BY. Micro-macroscopic coupled modeling of batteries and fuel cells. I. Model development. *J Electrochem Soc* 1998;145(Compendex):3407–17.
- [20] Wang CY, Cheng P. Multiphase flow and heat transfer in porous media. In: Hartnett James P, George AG, editors. *Advances in heat transfer*. Elsevier; 1997. TFIJYC, p. 93–182, a, 3–96.
- [21] ESI-CFD-Inc. CFD-ACE+ user manual. 4th ed. Huntsville, AL: ESI Group; 2009.
- [22] Tanaka M, Yamashita I, Chisaka F. Flow and heat transfer characteristics of Stirling engine regenerator in oscillating flow. *Nippon Kikai Gakkai Ronbunshu, B Hen/Trans Jpn Soc Mech Eng, Part B* 1989;55(516):2478–84.
- [23] Lemmon EW, Peskin AP, McLinden MO, Friend DG. NIST12. NIST thermodynamic and transport properties of pure fluids – NIST pure fluids. 5. National Institute of Standards and Technology; 2000.
- [24] Ashwin TR, Narasimham GSVL, Jacob S. CFD analysis of high frequency miniature pulse tube refrigerators for space applications with thermal non-equilibrium model. *Appl Therm Eng* 2010;30(2–3):152–66.
- [25] Antao DS, Farouk B. Experimental and numerical investigations of an orifice type cryogenic pulse tube refrigerator. *Appl Therm Eng* 2013;50(1):112–23.
- [26] Antao DS, Farouk B. Numerical simulations of transport processes in a pulse tube cryocooler: effects of taper angle. *Int J Heat Mass Transfer* 2011;54(21–22):4611–20.
- [27] Antao DS, Farouk B. Computational fluid dynamics simulations of an orifice type pulse tube refrigerator: effects of operating frequency. *Cryogenics* 2011;51(4):192–201.
- [28] Aktas MK, Farouk B. Numerical simulation of acoustic streaming generated by finite-amplitude resonant oscillations in an enclosure. *J Acoust Soc Am* 2004;116(5):2822–31.